MECE E6102 Computational Heat Transfer and Fluid Flow

(Spring 2023)

Course Meeting Times and Format

Lectures: 1 session/week, Wed 4:10-6:40 pm, Mudd 1024.

Office Hours

Prof. Vedula: by appointment, vv2316@columbia.edu
Course Assistant: TBA

Course Objectives and Outcomes

- Learn techniques to numerically simulate viscous fluid flows and lay the foundation for performing fluid-structure interaction (FSI).
- Gain requisite knowledge to choose the most suitable approach for developing a solver for a given flow configuration.

Prerequisites

- Any introductory fluid mechanics course (MECE E3100, MECE E4100, or equivalent).
- Any course on numerical methods (APAM E4300, E4301, or equivalent), or any course on computational fluid dynamics (CFD) or finite element method (FEM) including but not limited to CHEN E4150, ENME E4332, MECE E6104, or equivalent.
- Knowledge of a programming language (C, C++, Fortran, Python, etc.). MATLAB is acceptable.
- Instructor's permission is needed if any of the above requirements are not satisfied.

Description

Solving Navier-Stokes equations for incompressible viscous fluid flows using finite differences and finite element methods. Foundational concepts of CFD, including discretization, stability analysis, error dynamics, and variational principles. Special topics include methods for solving complex fluid flows, including immersed boundary methods and the arbitrary Lagrangian-Eulerian approach for performing fluid-structure interaction.

Textbooks

For basic CFD and scientific computing, I'd recommend,

- Computational Fluid Mechanics & Heat Transfer (Tannehill, Anderson and Pletcher, *CRC*)
- Computational Methods for Fluid Flow (Peyret and Taylor, *Springer*)
- Numerical Computation of Internal and External Flows, vols. 1 and 2 (Hirsch, *Wiley*)
- Fundamentals of Engineering Numerical Analysis (Moin, *Cambridge*)
- High Accuracy Computing Methods: fluid flow and wave phenomena (Sengupta, *Cambridge*)
- Computational Methods for Fluid Dynamics (Ferziger, Peric and Street, *Springer*)
- Computational Fluid Dynamics (John Anderson Jr., *McGraw-Hill*)

For the finite element method and fluid-structure interaction, I'd recommend,

- The Finite Element Method: linear static and dynamic finite element analysis (Hughes, *Dover*)
- The Finite Element Method for Fluid Dynamics (Zinkiewicz, Taylor, and Nithiarasu, *BH*)
- Computational Fluid-Structure Interaction (Bazilevs, Takizawa and Tezduyar, *Springer*

Certain books and journal articles will be recommended for specific topics during the course.

Preparing for this Course

- Review the fundamentals of numerical methods and fluid mechanics.
- Because this is a computational course with assignments requiring programming, please review basic programming in a language of your choice. Install necessary software, including compilers, dependencies, packages (for Python), MATLAB, FEniCS, etc.

Grading

- Homework (30%)
- Midterm (30%)
- Final Project (40%)

Late Homework Policy: Late HW is not accepted without prior permission from the instructor.

Topics Covered

- Review of discrete methods for PDE (finite differences, finite volume, and finite elements)
- Properties of the Navier-Stokes equations
- Numerical techniques for solving wave equation, linear and nonlinear Burger's equations
- Error analysis
- Solution techniques for solving the Navier-Stokes equations
- Flow simulation in complex geometries - immersed boundary methods
- Advanced topics include flow simulation in complex geometries using immersed boundary methods (IBM), and fluid-structure interaction (FSI) using Arbitrary Lagrangian-Eulerian (ALE) formulation

Academic Integrity

The strength of the university depends on academic and personal integrity. Ethical violations include cheating on exams, plagiarism, reuse of assignments (this includes copying from assignments of previous years as well as handing down assignments/exams to students who might take this course in the future), improper use of the Internet and electronic devices, unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition. While seeking help/advice in clarifying underlying concepts is OK, collaboration on HW assignments/projects is <u>NEVER</u> OK (unless specifically allowed by the instructor).

You can find more information about university misconduct policies on the web at these sites:
https://www.cc-seas.columbia.edu/integrity
https://www.cc-seas.columbia.edu/integrity/policy

Weekly Syllabus:

Week 1 (Jan 18): Review of key concepts from numerical methods - 1

- Discretization
- Consistency, stability, and convergence
- Aliasing error
- Conservation principles
- Finite volume method: discretization on Cartesian grid

Week 2 (Jan 25): Review of key concepts from numerical methods - 2

- Finite volume method: discretization on non-Cartesian grids
- Conservation errors
- Navier-Stokes equation and its properties

Week 3 (Feb 1): Review of Iterative Methods

- Initial and boundary value problems
- Properties: smoothing, min-max
- Iterative methods
  - Point-Jacobi
  - Gauss-Seidel
  - Over-relaxation techniques
  - Krylov-subspace methods (steepest descent, conjugate gradients)

Week 4 (Feb 8): Finite difference solution of viscous Burger's equation

- Burger's equation as a model problem for Navier-Stokes
- Linear and nonlinear variants
- Modified wave-number analysis
- Von-Neumann stability analysis
- Effects of cell Peclet number
- Viscous stability constraint
- Implicit and explicit discretizations

Week 5 (Feb 15): Finite difference solution of Navier-Stokes equations

- Newton-Raphson for nonlinear Burgers equation
- Navier-Stokes: overconstrained, mass conservation
- Pressure-Poisson equation (PPE)
- Procedure to solve Navier-Stokes equations
- Challenges:
  - Appropriate treatment of pressure
  - Pressure-velocity decoupling problem
  - Compact vs. non-compact stencil
  - Checkered-board problem

Week 6 (Feb 22): Overcome challenges to solving Navier-Stokes

- Grid-staggering, filtering, upwinding
- Compatibility constraint
- Fractional step method (operator splitting)
- Appropriate treatment of boundary conditions
- Staggered schemes

Week 7 (Mar 1): Other solution procedures

- Hybrid schemes: intermediate between collocated and staggered approaches
- Other techniques: SIMPLE, artificial compressibility method
- Discussion of the final project (lid-driven cavity flow)

Week 8 (Mar 8): The Finite Element Method

- The FEM approach: strong/weak forms, Galerkin/matrix forms
- Piecewise linear finite elements

Take-Home Midterm Exam due after Spring Break

Week 9 (Mar 13 – Mar 17) Spring Break

Week 10 (Mar 22): FEM for the convection-diffusion equation

- Galerkin discretization of the 1D convection-diffusion equation
- Oscillatory solutions, dependence on cell Peclet number
- Exact artificial diffusion (EAD) method
- Conservative vs. convective form

Week 11 (Mar 29): Stabilized FEM

- Galerkin FEM: element point-of-view
- Equivalence with central difference approximation
- Galerkin method for EAD method, inconsistency
- Concept of stabilization to restore consistency
    - The SUPG method

Week 12 (Apr 5): Multi-dimensions and Variational Multiscale (VMS) method

- SUPG for 2D/3D convection-diffusion equation
- Variational Multiscale Method
- Navier-Stokes in arbitrary Lagrangian-Eulerian (ALE) formulation

Week 13 (Apr 12): FEM for Navier-Stokes

- Mixed FEM
- Inf-sup conditions and incompressibility constraints
- ALE-VMS formulation for Navier-Stokes
- Generalized-alpha method for time-integration

Week 14 (Apr 19): Immersed Boundary Methods (IBM)

- Overview of grid generation
- Classical IBM (Peskin's method)
- Sharp-Interface IBM
- Immersed Finite Element Method (IFEM)

Week 15 (Apr 26): Introduction to Fluid-Structure Interaction (FSI)

- Body-conforming or interface-tracking methods (ALE)
- Non-body conforming or interface-capturing methods (IBM)

Week 16 (May 1 – May 5): Study Week

Week 17 (May 10): Final Project Due