MECE E6106 Finite Element Method for Fluid Flow and Fluid-Structure Interactions

(Fall 2021)

Course Meeting Times and Format

Lectures: 1 session / week, Wed 4:10-6:40pm, Mudd 337/Hybrid.

Office Hours

Prof. Vedula: by appointment, vv2316@columbia.edu
Course Assistant: TBA

Course Objectives and Outcomes

- Learn to apply the finite element method to simulate viscous fluid flows and fluid-structure interaction (FSI).

Prerequisites

- Any graduate-level fluid mechanics course (MECE E4100, MECE E6100, or equivalent).
- Any introductory course on finite element method (ENME E4332), or equivalent.
- Knowledge of high-level programming language (C, C++, Fortran, MATLAB, Python, etc.)
- Instruction's permission is needed if any of the above requirements is not satisfied.

Description

Solving convection-dominated phenomena using finite element method (FEM) including convection-diffusion equation, Navier-Stokes equation for incompressible viscous flows, and nonlinear fluid-structure interaction (FSI). Foundational concepts of FEM include function spaces, strong and weak forms, Galerkin FEM, isoparametric discretization, stability analysis, and error estimates. Mixed FEM for Stokes flow, incompressibility and inf-sup conditions. Stabilization approaches including residue-based variational multiscale methods. Arbitrary Lagrangian-Eulerian (ALE) formulation for nonlinear FSI, and selected advanced topics of research interest.

Textbooks

For the finite element method and fluid-structure interaction, I'd recommend,

- The Finite Element Method: linear static and dynamic finite element analysis (Hughes, *Dover*)
- Nonlinear Finite Element Methods (Peter Wriggers, Springer)
- Computational Fluid-Structure Interaction (Bazilevs, Takizawa and Tezduyar, *Springer*)
- The Finite Element Method for Fluid Dynamics (Zinkiewicz, Taylor and Nithiarasu, *BH*)

Certain books and journal articles will be recommended for specific topics during the course.

Preparing for this Course

- Review the fundamentals of finite element method (Chapters 1-3 from the book by *Hughes*).
- Review the basics of fluid mechanics from any graduate-level text book (e.g., Fluid mechanics by Kundu, Cohen & Dowling (Academic Press), Incompressible Flows by Panton (Wiley), etc.)
- Please note that this is a computational course with assignments requiring programming. Kindly review basic programming skills in a language of your choice. Install necessary software including compilers, dependencies, packages (for Python), MATLAB, etc.

Grading

- Homework (60%)
- Project (40%)

Late Homework Policy: Late HW is not accepted without prior permission from the instructor.

Syllabus

- Review of the basics of FEM (function spaces, strong/weak form, standard Galerkin method, isoparametric discretization)
- Convection-diffusion equation as the model problem
- Stability and Error analysis
- Stabilization methods including SUPG, variational multiscale methods
- The Stokes flow: mixed FEM, inf-sup conditions, and incompressibility
- PSPG stabilization
- Stabilization methods for the Navier-Stokes equations
- Review of nonlinear solid mechanics
- The arbitrary Lagrangian-Eulerian (ALE) formulation for FSI
- Time integration methods
- Review of linear algebra for FEM (preconditioners, direct and iterative methods)
- Advanced topics include bipartition method for incompressible flows, mixture theory-based formulation for fluid flows and FSI, and a unified formulation for fluid/solid mechanics and FSI

Academic Integrity

The strength of the university depends on academic and personal integrity. Ethical violations include cheating on exams, plagiarism, reuse of assignments (this includes copying from assignments of previous years as well as handing down assignments/exams to students who might take this course in the future), improper use of the Internet and electronic devices, unauthorized collaboration, alteration of graded assignments, forgery and falsification, lying, facilitating academic dishonesty, and unfair competition. While seeking help/advice in clarifying underlying concepts is OK, collaboration on HW assignments/projects is NEVER OK (unless specifically allowed by the instructor).